# Late-time entropy production due to the decay of domain walls

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It is shown that late-time decay of domain walls can dilute unwanted relics such as moduli, if the universe was dominated by frustrated domain walls with tension $\sigma = (1 \sim 100\,\mathrm{TeV})^3$. Since energy density of the frustrated domain walls decreases as slow as the inverse of the scale factor, an overclosure limit on the axion decay constant $f_a$ is also considerably relaxed. In fact $f_a$ can be as large as the Planck scale, which may enable us to naturally implement the QCD axion in the string scheme. Furthermore, in contrast to thermal inflation models, the Affleck-Dine baryogenesis can generate enough asymmetry to explain the present baryon abundance, even in the presence of late-time entropy production.

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Particle physics beyond the standard model predicts a number of new particles, some of which have long lifetimes and decay at cosmological time scales. If such long-lived particles are substantially produced in the early universe, they may give crucial effects on cosmology and spoil success of the standard big-bang model.

One well-known example of such dangerous relic particles is gravitino in supergravity theories. Gravitinos are produced during reheating after inflation and the abundance of the produced gravitino is proportional to the reheating temperature. The gravitino with mass $\sim 0.1 - 10$ TeV decays during or after the big-bang nucleosynthesis (BBN) and may destroy the light elements synthesized in BBN. Thus, in order to keep success of BBN the reheating temperature should be sufficiently low (for recent works, see [1, 2, 3, 4]).

Another class of dangerous relics are light scalar fields called moduli which appear in superstring theories [5, 6, 7]. The moduli fields are expected to acquire masses of the order of the gravitino mass from nonperturbative effects of the SUSY breaking and have long lifetimes since they have only the gravitationally suppressed interactions. During inflation the field value of moduli generally deviates from the vacuum value due to extra SUSY breaking by the cosmic density. The deviation is of the order of the Planck scale $M_G (= 2.4 \times 10^{18}$ GeV) since no other mass scale exists. Then, when the Hubble becomes less than the modulus mass, the modulus field starts to osciilate with amplitude $\sim M_G$. Because the modulus denity is comparable to the cosmic density at onset of the oscillation, it soon dominates the density of the universe and causes the serious cosmological problem (moduli problem). Since the abundance of the moduli is independent of the reheating temperature, the moduli problem is much more serious than the gravitino problem. To solve the moduli problem (as well as the gravitino problem) it is necessary to dilute the oscillating moduli by huge entropy production. So far the most successful model to produce such large entropy is thermal inflation proposed by Lyth and Stewart [8, 9]. The thermal inflation is mini-inflation with the number of e-folds $\sim 10$ which takes place at the weak scale. A comprehensive study of the thermal inflation was done in Ref. [10] and it was shown that for modulus mass between 10 eV and 10 TeV the thermal inflation solves the cosmological moduli problem. However, the thermal inflation also dilute the baryon number of the universe and hence we need an efficient baryogenesis mechanism. In Refs. [10, 11] it was pointed out that the Affleck-Dine mechanism does that job. But later it was shown that $Q$-ball formation obstructs the baryogenesis [12].

In this letter we propose an alternative model to dilute the dangerous cosmological relics. In our model domain walls, which are usually considered as cosmological disaster, play an important role. The domain walls are produced when some discrete symmetry is spontaneously broken. The domain wall network in the universe becomes very complicated if there are many discrete vacua. The density of such domain wall network decreases very slowly and quickly dominates the universe. In extreme case, the domain walls become frustrated and their density $\rho_{DW}$ decreases as $\rho_{DW} \propto a^{-1}$ ($a$: the scale factor) which is slower than the scaling evolution $\rho_{DW} \propto a^{-3/2}$ in matter-dominated universe. Furthermore, if the scalar potential has a tiny term which explicitly breaks the discrete symmetry, the vacua separated by domain walls have slightly different energies and hence the domain wall network is no longer stable and decays producing large entropy. It will be shown that the entropy production by the decay of domain wall is enough to dilute the moduli density. Moreover, because the density of domain wall decreases much more slowly than the matter density, the cosmic axion density is also diluted. As a result, the axion decay constant $f_a$ as large as $M_G$ is allowed, which might enable us to implement the axion into the string framework. In addition, the Affleck-Dine baryogenesis is compatible with our model as we will see below. Therefore, the domain walls provide more efficient and consis-

tent dilution mechanism than the thermal inflation.

First we present our scenario that the decay of domain walls generates large entropy to dilute unwanted relics such as moduli and gravitinos, and derive constraints on tension of the domain walls. In contrast to the other candidates for late-time entropy production, domain walls have an advantage that the energy density decreases relatively slowly, which ensures huge entropy production. To be specific, an equation-of-state $w$ equals to $-2/3$ if the domain walls are completely frustrated, that is, the structure of the domain wall network remains unchanged as the universe expands. Throughout this letter, we assume this is the case. Later we give a model which would lead to such frustrated domain walls.

Let us consider the evolution of the energy density of domain walls, which eventually dominate the universe. The energy density of the domain walls at the formation is estimated as

$$\rho_{DW,i} \sim \sigma H_i, \tag{1}$$

where $\sigma$ is the tension of the wall, $H_i$ the Hubble parameter at the phase transition. We have assumed that there are only a few domain walls per one horizon when they are formed. Here and in what follows we neglect $O(1)$ numerical factors. If the domain walls are fully frustrated, the energy density falls off as the inverse of the scale factor: $\rho_{DW} \propto a^{-1}$. For definite discussion, we assume that the universe is dominated by either the inflaton or modulus fields when the domain walls are formed. Noting that the energy density of an oscillating massive scalar field decreases as $\propto a^{-3}$, the domain walls dominate the universe when the Hubble parameter becomes equal to

$$H_{eq} \sim \sigma^{\frac{3}{4}} H_i^{\frac{1}{4}} M_G^{-\frac{3}{2}}. \tag{2}$$

If the reheating occurs earlier than $H = H_{eq}$, the radiation-dominated epoch might last for a while, which makes it easy for the domain walls to dominate the unverse. Thus, for conservative discussion, we assume that the universe is dominated by nonrelativistic particles until the domain walls dominate the universe.

The decay of domain walls can proceed if there is a tiny bias between vacua, $\delta\rho$. When this energy difference becomes comparable to the energy of the domain walls, the decay occurs and the universe is reheated. Since the universe was dominated by the domain walls in our scenario, $\delta\rho$ is simply related to the decay temperature as $\delta\rho \sim T_d^4$. In order to have both large enough entropy production and the successful BBN, we take the decay temperature as low as 10MeV. Note that this value is just an exemplified value and that the successful dilution is actually realized for a wide range of the decay temperature, say, $T_d = 10\text{MeV} \sim 10\text{GeV}$ (see Eqs. (3), (4) and (7)). It is the smallness of $\delta\rho$ that enables domain walls to be long-lived. Let us note that, when the domain walls decay, at least several domain walls must be present inside one horizon, otherwise the old inflation occurs somewhere leading to unacceptably inhomogeneous universe. This requires that the decay temperature $T_d$ satisfy the following inequality: $T_d^4 > \sigma H_d$, where $H_d \sim T_d^2/M_G$ is the Hubble parameter when the domain walls decay. Thus the tension is bounded above:

$$\sigma < T_d^2 M_G \sim (100\,\text{TeV})^3 \left(\frac{T_d}{10\,\text{MeV}}\right)^2. \tag{3}$$

On the other hand, the tension should be large enough to dominate the universe before the decay: $H_{eq} > H_d$. That is,

$$\sigma > \left(\frac{T_d^8 M_G^2}{H_i}\right)^{\frac{1}{3}} \sim (100\,\text{GeV})^3 \left(\frac{T_d}{10\,\text{MeV}}\right)^{\frac{8}{3}} \left(\frac{H_i}{1\,\text{TeV}}\right)^{-\frac{1}{3}}. \tag{4}$$

Now let us estimate the the abundance of the modulus field after decay of the domain walls. For domain walls with the tension satisfying (3) and (4), $H_{eq}$ is much smaller than the modulus mass. Since the modulus field dominates the universe as soon as it starts oscillating, it is reasonable to assume that the universe was dominated by the modulus field when $H = H_{eq}$. Then the modulus-to-entropy ratio is given by

$$\frac{\rho_{mod}}{s} \sim T_d \left(\frac{H_d}{H_{eq}}\right)^4 \sim \frac{T_d^9 M_G^2}{\sigma^3 H_i}, \tag{5}$$

where we have used the fact that the Hubble parameter falls off as $\propto a^{-1/2}$ while the domain walls dominate the universe, and Eq. (2) is substituted in the last equation. This must satisfy the observational bound:

$$\frac{\rho_{mod}}{s} < \kappa \frac{\rho_c}{s} = 3.6 \times 10^{-9} \kappa\, h^2\,\text{GeV}, \tag{6}$$

where $\rho_c$ is the critical density, $\kappa$ varies from $10^{-10}$ to 0.2 depending on the modulus mass [10], and $h$ is the Hubble constant in units of 100 km/sec/Mpc. Thus the tension is further constrained as

$$\sigma > \kappa^{-\frac{1}{3}} (500\,\text{GeV})^3 \left(\frac{T_d}{10\,\text{MeV}}\right)^3 \left(\frac{H_i}{1\,\text{TeV}}\right)^{-\frac{1}{3}}. \tag{7}$$

To sum up, the domain walls with the tension $\sigma = (1 \sim 100\,\text{TeV})^3$, which decay just before the BBN starts, can dilute the moduli to the observationally allowed level.

Next we present a model that would lead to formation of the frustrated domain walls. Let us consider the following superpotential:

$$W = \sqrt{\lambda} \sum_i^N \sum_j^N Z_{ij} \Phi_i \Phi_j + \frac{2\epsilon}{\sqrt{\lambda}} \sum_i Z_{ii} \Phi_i^2, \tag{8}$$

where $Z_{ij}$ and $\Phi_i$ are gauge-singlet superfields, and real coupling constants $\lambda > 0$ and $\epsilon$ are assumed to satisfy the

inequality, $\lambda \gg |\epsilon|$ [17]. These interactions respect $S_N$ symmetry if $\Phi_i$ and $Z_{ij}$ are taken to be the fundamental and bi-fundamental representations of $S_N$ permutation group. The $R$-charges are assigned as $R_\Phi = 0$ and $R_Z = 2$. We assume that $Z_{ij}$ always sits at the origin: $\langle Z_{ij} \rangle = 0$, which can be realized if $Z_{ij}$ acquires a positive Hubble-induced mass term during relevant epoch.

Then we obtain the following effective potential for $\Phi_i$,

$$\begin{aligned} V(\Phi) \; \simeq \; & V_0 - m_0^2 \sum_i |\Phi_i|^2 - m_{3/2}^2 \sum_i (\Phi_i^2 + \Phi_i^{*2}) \\ & + \lambda (\sum_i |\Phi_i|^2)^2 + 4\epsilon \sum_i |\Phi_i|^4, \end{aligned} \tag{9}$$

where $m_{3/2}$ is the gravitino mass, $V_0 = O(m_0^4/\lambda)$ is chosen in such a way that the cosmological constant vanishes in the true minimum, and we have used $\lambda \gg |\epsilon|$. Here we have assumed the negative mass of the order of the weak scale at the origin, $m_0 \sim \Lambda_{EW} \sim 1\,\mathrm{TeV}$, which is induced by the SUSY breaking effects. The third term comes from the gravity-mediated SUSY breaking effects. Due to this term, the global minima of the potential lie along the real axes of $\Phi_i$. In order to study the vacuum structure of this potential, let us concentrate on the the real components of $\Phi_i$ by setting $\mathrm{Im}\,\Phi_i = 0$. Rewriting the potential in terms of the real components $\phi_i \equiv \sqrt{2}\,\mathrm{Re}\,\Phi_i$, we obtain

$$V(\phi) \simeq V_0 - m_0^2 \sum_i \phi_i^2/2 + \lambda(\sum_i \phi_i^2)^2/4 + \epsilon \sum_i \phi_i^4, \tag{10}$$

where the third term in Eq. (9) is neglected since we are interested in the case of $m_{3/2} \lesssim m_0$. The scalar potential given by Eq. (10) actually agrees with the $O(N)$ model studied in Refs. [13, 14]. The approximate $O(N)$ symmetry originates from the hierarchical couplings, $\lambda$ and $\epsilon$. Note that this potential is obtained by disregarding $\mathrm{Im}\,\Phi_i$, so it does not necessarily give a right answer, for instance, when one estimates the energy density inside domain walls, although it is still useful to study the vacuum structure.

The positions of the global minima of $V(\phi)$ depend on the sign of $\epsilon$. For $\epsilon < 0$, there are $2N$ minima:

$$\phi_{\rm min}^{\pm\,(i)} = (0, \ldots, 0, \pm v_1, 0, \ldots, 0), \qquad \text{for} \quad i = 1 \sim N \tag{11}$$

with $v_1 \equiv m_0/\sqrt{\lambda}$. On the other hand, if $\epsilon > 0$, the potential minima are given by

$$\phi_{\rm min} = (\pm v_2, \ldots, \pm v_2) \tag{12}$$

with $v_2 \equiv m_0/\sqrt{\lambda N}$, where arbitrary combination of the signs are allowed. In the following, we consider the case of $\epsilon > 0$. Then all $Z_{ij}$ fields acquire masses of the order of $v_2 \sim \Lambda_{EW}$ in these minima. Also, the number of the minima is $2^N$. Thus, for large $N$, the vacuum structure becomes more complicated, compared to the former case. Since there are many types of vacua, pair annihilation processes of walls are expected to be highly suppressed, which enables walls to deviate from the scaling law. The minima are separated by the potential barrier represented by the third term in Eq. (9), if $\epsilon$ is larger than $m_{3/2}^2/v_2^2 \sim \lambda N m_{3/2}^2/m_0^2$. Then the tension of the walls is given by

$$\sigma_1 \sim m_{3/2} v_2^2 \sim (1\,\mathrm{TeV})^3\,(\lambda N)^{-1} \left(\frac{m_{3/2}}{1\,\mathrm{TeV}}\right) \left(\frac{m_0}{1\,\mathrm{TeV}}\right)^2. \tag{13}$$

On the other hand, if $\epsilon$ is smaller than $m_{3/2}^2/v_2^2$, the potential barrier is the last term in Eq. (9). The tension is $\sigma_2 \sim |\epsilon|^{\frac{1}{2}} v_2^3 \leq \sigma_1$, where the last equality holds when $\epsilon \sim m_{3/2}^2/v_2^2$. Thus the tension of the domain walls in this model can satisfy the bound derived in the previous section, although $\lambda N \lesssim 1$ might be necessary in the latter case.

We assume that all $\Phi_i$ get positive Hubble-induced masses therefore sit at the origin until the Hubble parameter becomes comparable to $m_0$. Then those scalar fields start rolling down and get settled somwhere on $S^{N-1}$ defined by $\sum_i |\Phi_i|^2 = 2Nv_2^2$. When the Hubble parameter becomes equal to the gravitino mass, all $\Phi_i$ move to the real axes due to the third term in Eq. (9) and fall in one of the vacua (12). Then the domain walls with the tension $\sigma_1$ or $\sigma_2$ are formed.

Domain walls must decay before the relevant BBN epoch. To this end, we introduce a R-symmetry violating interaction that lifts the degeneracy of the vacua: $W = c' \langle W \rangle \sum_i \Phi_i^3/M_G^3$ with $\langle W \rangle = m_{3/2} M_G^2$, leading to $V_A = c\, m_{3/2}^2 \sum_i \Phi_i^3/M_G$+h.c., where $c$ and $c'$ are coupling constants. For simplicity we take $c$ both real and negative so that true minimum is given by $\phi_{\rm true\,min} = (v_2, \ldots, v_2)$. The decay temperature is then expressed by

$$\frac{T_d}{100\mathrm{MeV}} \;\sim\; (\lambda N)^{-\frac{3}{8}} \left(\frac{c}{0.1}\right)^{\frac{1}{4}} \left(\frac{m_{3/2}}{1\mathrm{TeV}}\right)^{\frac{1}{2}} \left(\frac{m_0}{1\mathrm{TeV}}\right)^{\frac{3}{4}} \tag{14}$$

When the bias becomes comparable to the energy of the domain walls, the topological defects are no longer stable and decay into $\Phi$-particles in the true vacuum. Not to spoil the success of the BBN, however, $\Phi$-particles should decay into standard model degrees of freedom. We assume that $\Phi_i$ weakly couples to some heavy vector-like quarks, which enable $\Phi$-particles to radiatively decay into the standard model gauge bosons as soon as the domain walls decay. Since the $R$-parity of $\Phi$ is even, the decay processes into the lightest supersymmetric particles (LSPs), which may overclose the universe, can be avoided if the mass of $\Phi$ is smaller than two times the LSP mass. Thus our model of domain walls can naturally satisfy the constraints necessary to induce successful dilution.

As we saw in the above discussions, the domain wall can be a viable candidate for late-time entropy production. What differs from the other candidates is that the energy density falls off very slowly: $\rho_{DW} \propto a^{-1}$. Such a distinctive feature can lead to another important cosmological consequence: the overclosure limit on the axion decay constant can be considerably relaxed. If we do not

assume entropy production after axion begins the coherent oscillation, the axion decay constant is constrained as $F_a \lesssim 10^{12}$GeV not to overclose the universe. This upperlimit is relaxed to $10^{15}$ GeV, if the late-time entropy production due to the decay of nonrelativistic particles occurs [15]. In the following, we show that the axion decay constant is further relaxed and can be as large the Planck scale if the domain-wall induced entropy production occurs well below the QCD scale.

The axion starts to oscillate when the Hubble parameter becomes comparable to the mass: $3H_{\rm osc} \simeq m_a(T_{\rm osc})$. The axion mass $m_a$ depends on the temperature $T$ as [16]

$$m_a(T) \simeq \begin{cases} 0.1 m_a (\Lambda_{\rm QCD}/T)^{3.7} & \text{for } T \gtrsim \Lambda_{\rm QCD}/\pi, \\ m_a & \text{for } T \lesssim \Lambda_{\rm QCD}\pi, \end{cases} \tag{15}$$

where $\Lambda_{\rm QCD} \simeq 0.2$GeV, and $m_a \simeq \Lambda_{\rm QCD}^2/F_a$ is the axion mass at the zero temperature. Since the number density of the axion decreases as $\propto a^{-3}$, the energy density of the axion when the domain walls decay is

$$\rho_a|_{T=T_d} = \frac{1}{2} m_a \left(m_a(T_{\rm osc}) F_a^2 \theta^2\right) \frac{H_d^6}{H_{\rm osc}^6}, \tag{16}$$

where we have used $H \propto a^{-1/2}$ when the domain walls dominate the universe. $\theta \sim O(1)$ denotes the initial amplitude of the axion field in the unit of $F_a$. The axion-to-entropy ratio is then given by

$$\frac{\rho_a}{s} = 1.3 \times 10^2 \frac{F_a^6\, \theta^2\, T_d^9}{\xi(T_{\rm osc})^5 \Lambda_{\rm QCD}^8 M_G^6}, \tag{17}$$

where $\xi(T) \equiv m_a(T)/m_a \leq 1$. This must be smaller than the present value of the ratio of the critical density to the entropy. That is, the axion decay constant should satisfy

$$\begin{aligned} F_a \;\lesssim\; & 3.6 \times 10^{18}\,{\rm GeV}\, \xi(T_{\rm osc})^{\frac{5}{6}} \theta^{-\frac{1}{3}} \\ & \times \left(\frac{\Omega_a h^2}{0.14}\right)^{\frac{1}{6}} \left(\frac{\Lambda_{\rm QCD}}{0.2\,{\rm GeV}}\right)^{\frac{4}{3}} \left(\frac{T_d}{10\,{\rm MeV}}\right)^{-\frac{3}{2}} . \end{aligned} \tag{18}$$

Therefore the domain-wall induced entropy production opens up the axion window to the Planck scale, if $\xi(T_{\rm osc})$ is close to 1. In other words, the axion is a good candidate for dark matter if $F_a \sim M_G$. We need to check that $T_{\rm osc}$ is smaller than 0.1 GeV so that $\xi(T_{\rm osc})$ is $\sim 1$. To this end, the evolution of the cosmic temperature before the decay of domain walls must be specified. If the decay is approximated to be an exponential decay with a constant decay rate, we obtain

$$T_{\rm osc} \simeq 0.05{\rm GeV} \left(\frac{T_d}{10\,{\rm MeV}}\right)^{0.26} \left(\frac{F_a}{M_G}\right)^{-0.13} \lesssim 0.1\,{\rm GeV}. \tag{19}$$

Therefore, $\xi(T_{\rm osc})$ is close to 1 when $T_d \sim 10\,$MeV and $F_a \sim M_G$.

In this letter we have investigated the dilution of unwanted cosmological relics such as moduli by entropy production of the domain wall network. It has been shown that the frustrated domain walls quickly dominate the density of the universe and their decay produces entropy large enough to dilute the dangerous moduli. We have also given a concrete model which leads to frustrated network of domain walls. Moreover, we have found that the late-time decay of the domain walls greatly relaxes the constraint on the axion model and the axion decay constant $f_a$ as large as the Planck scale is allowed.

Up to here, we have assumed fully frustrated domain walls, however, this assumption can be weaken to some extent. Similar arguments show that domain walls whose energy evolves as $\rho_{DW} \propto a^{-1-\gamma}$ with $\gamma \lesssim 0.2(0.1)$ for $\kappa = 0.2(10^{-10})$ works as well. Finally we make a comment on baryon number generation in the present model. The domain-wall decay also dilute pre-existing baryon number. Therefore, there should be large baryon asymmetry before the entropy production or baryon number should be generated after the decay of the domain wall. Since the decay temperature is expected to be low ($\sim 10$ MeV), it is unlikely to produce the baryon number after the decay. Then the most promising candidate for baryogenesis is the Affleck-Dine mechanism. In order to produce sufficient baryon asymmetry the gravitino mass should be relatively small, $m_{3/2} \lesssim 10$ MeV [10], as in the gauge-mediated SUSY breaking scenarios [18]. In the case of the thermal inflation, the Affleck-Dine mechanism does not work due to $Q$-ball formation [12]. The crucial point is that the thermal inflation requires large messenger scale, which leads to large $Q$-balls and small baryon number in the background universe. However, our model allows relatively small messenger scale, since $v_2$ is much smaller than the vev of the flaton. Thus the Affleck-Dine baryogenesis does work in the present scenario.

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[17] Note that large hierarchy between $\lambda$ and $\epsilon$ is not necessary, since the tension of the domain walls is independent of $\epsilon$ as long as $\epsilon$ is not too small. In fact, $\epsilon \sim 0.1\lambda$ does work.
[18] For such small $m_{3/2}$, $\lambda$ ($m_0$) should be so small (large) that the tension is larger than $(1\,\mathrm{TeV})^3$. See Eq. (13).